Stackelberg-Pareto Synthesis and Verification

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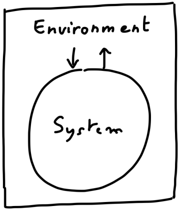
Joint work with Jean-François Raskin and Clément Tamines

Reactive synthesis

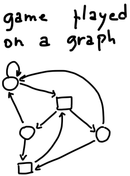
Reactive systems

- System which constantly interacts with an uncontrollable environment
- It must satisfy some property against any behavior of the environment
- How to automatically design a correct controller for the system?



Modelization

- Two-player zero-sum game played on a finite directed graph
- Property = objective for the system
- Synthesis of a controller = construction of a winning strategy



Reactive synthesis

Classical approach with numerous results and several tools, see e.g.

- The book chapter "Graph Games and Reactive Synthesis" [BCJ18]
- My survey "Computer Aided Synthesis: a Game Theoretic Approach" in the Proceedings of DLT 2017 [Bru17]

Disadvantages

Fully adversarial environment: bold abstraction of reality

- Assumes the only goal of the environment is to make the system fail
- Environment can be composed of one or several components, each with its own objective

More adequate models

Stackelberg games: non zero-sum games

- System: a specific player called the leader
- Environment: composed of the other players called followers
- The leader first announces his strategy and then the followers respond by playing rationally given that strategy
- The leader wants to satisfy his objective whatever the rational response of the followers

In the next slides

- One follower: presentation of the new model proposed in [BRT21] and the obtained results [BRT21, BRT22]
- Several followers: some results presented at the end of the talk

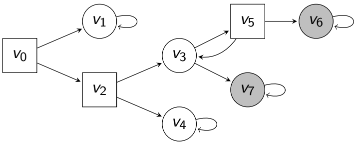
Stackelberg-Pareto games

Definitions

- Game arena: graph $G = (V, V_0, V_1, E, v_0)$
  with (V_0, V_1) a partition of V and v_0 an initial vertex
- Two players: Player i that controls vertices of V_i, $i = 0, 1$
  Player 0 is the leader and Player 1 is the follower
- Play: infinite path starting from v_0
- Objective for Player i: subset Ω of plays
  A play ρ satisfies Ω if $\rho \in \Omega$

Example

- Player 0: circle vertices
- Player 1: square vertices
- Objective Ω_0 of Player 0:
  reach $\{v_6, v_7\}$



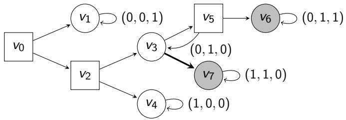
Stackelberg-Pareto games

Definitions

- Stackelberg-Pareto game: $\mathcal{G} = (G, \Omega_0, \Omega_1, \ldots, \Omega_t)$ with objective Ω_0 for Player 0 and t objectives $\Omega_1, \ldots, \Omega_t$ for Player 1
- Strategy $\sigma_0 : V^* \times V_0 \to V$ announces the choices of Player 0 after each history hv with $v \in V_0$
- $Plays_{\sigma_0} = \{\text{plays } \rho \mid \rho \text{ consistent with } \sigma_0\}$
- Payoff of $\rho \in Plays_{\sigma_0}$ for Player 1: Boolean vector $pay(\rho) \in \{0,1\}^t$

Example

- Ω_0: reach $\{v_6, v_7\}$
- 3 objectives Ω_1, Ω_2, Ω_3
- Strategy σ_0: choice of $v_3 \to v_7$ after history $v_0 v_2 v_3$
- $Plays_{\sigma_0} = \{v_0 v_1^\omega, v_0 v_2 v_3 v_7^\omega, v_0 v_2 v_4^\omega\}$



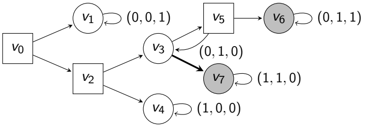
Stackelberg-Pareto games

Rationality of Player 1

- Componentwise order $<$ on the payoffs $pay(\rho) \in \{0,1\}^t$, $\forall \rho \in Plays_{\sigma_0}$
- Set P_{σ_0} of Pareto-optimal payoffs of $Plays_{\sigma_0}$ w.r.t. $<$
- Player 1 only responds with plays $\rho \in Plays_{\sigma_0}$ with a Pareto-optimal payoff $pay(\rho) \in P_{\sigma_0}$
- Goal of Player 0: announce σ_0 such that Ω_0 is satisfied by every such rational response

Example

- Ω_0: reach $\{v_6, v_7\}$
- $Plays_{\sigma_0} = \{v_0 v_1^\omega, v_0 v_2 v_3 v_7^\omega, v_0 v_2 v_4^\omega\}$
- $P_{\sigma_0} = \{(0,0,1), (1,1,0), \cancel{(1,0,0)}\}$



Stackelberg-Pareto verification

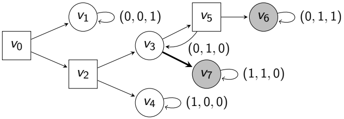
Given a strategy σ_0 announced by Player 0, verify whether or not his goal is satisfied

Stackelberg-Pareto verification problem (SPV problem)

Given a Stackelberg-Pareto game $\mathcal{G} = (G, \Omega_0, \Omega_1, \dots, \Omega_t)$ where the strategy of σ_0 of Player 0 is fixed, decide whether every play in $Plays_{\sigma_0}$ with a Pareto-optimal payoff satisfies the objective of Player 0

Example

- Ω_0: reach $\{v_6, v_7\}$
- $Plays_{\sigma_0} = \{v_0 v_1^\omega, v_0 v_2 v_3 v_7^\omega, v_0 v_2 v_4^\omega\}$
- $P_{\sigma_0} = \{(0,0,1), (1,1,0), \sout{(1,0,0)}\}$
- No, Ω_0 not always satisfied



Stackelberg-Pareto verification

Stackelberg-Pareto verification problem (SPV problem)

Given a Stackelberg-Pareto game $\mathcal{G} = (G, \Omega_0, \Omega_1, \ldots, \Omega_t)$ where the strategy of σ_0 of Player 0 is fixed, decide whether every play in $Plays_{\sigma_0}$ with a Pareto-optimal payoff satisfies the objective of Player 0

Theorem [BRT22]

The SPV problem is co-NP-complete for parity objectives, with a fixed-parameter algorithm (exponential in t)

Remarks

- Parity: a classical way to define ω-regular objectives (reachability, safety, Büchi, co-Büchi, Streett, Rabin, Muller, LTL, etc)
- Restriction to finite-memory strategies σ_0, i.e., described by a finite automaton
- Fixed-parameter complexity: in practice parameter t is small

Stackelberg-Pareto verification

Idea of the proof for co-NP membership

- Consider the complement of the SPV problem: does there exist a play in $Plays_{\sigma_0}$ with a Pareto-optimal payoff and not satisfying Ω_0?
- Algorithm
  - non-deterministically guess a payoff $p \in \{0,1\}^t$ (polynomial size)
  - check that there exists a play with payoff p (p is realizable)
  - check that there exists no play with a greater payoff (p is Pareto-optimal)
  - check that there exists a play with payoff p and not satisfying Ω_0
- The last three checks can be done in polynomial time (using automaton)
- Therefore in co-NP

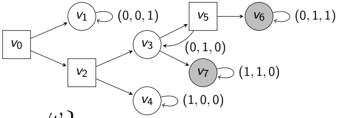
Problem

Stackelberg-Pareto Synthesis Problem (SPS problem)

Given a Stackelberg-Pareto game $\mathcal{G} = (G, \Omega_0, \Omega_1, \dots, \Omega_t)$, decide whether there exists a strategy σ_0 for Player 0 such that for every play $\rho \in Plays_{\sigma_0}$ with $pay(\rho) \in P_{\sigma_0}$, it holds that $\rho \in \Omega_0$

Example

- Yes, such a strategy σ_0 exists:
  - after $v_0 v_2 v_3$: $v_3 \to v_5$
  - after $v_0 v_2 v_3 v_5 v_3$: $v_3 \to v_7$
- $Plays_{\sigma_0} = \{v_0 v_1^\omega, v_0 v_2 v_3 v_5 v_6^\omega, v_0 v_2 v_3 v_5 v_3 v_7^\omega, v_0 v_2 v_4^\omega\}$
- $P_{\sigma_0} = \{$~~(0, 0, 1)~~$, (0, 1, 1), (1, 1, 0),$ ~~(1, 0, 0)~~$\}$



Results

Theorem [BRT21]

The SPS problem is NEXPTIME-complete for parity objectives, with a fixed-parameter algorithm (double exponential in t and exponentiel in the highest priorities)

Remark

- For reachability objectives, the SPS problem is NEXPTIME-complete and becomes NP-complete on tree arenas

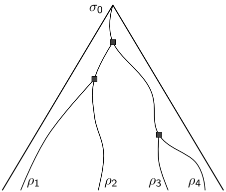
NEXPTIME-membership

Idea of the proof for NEXPTIME-membership

- If Player 0 has a solution σ_0 to the SPS problem, then he has a finite-memory one with an exponential size
- Algorithm
  - non-deterministically guess a strategy σ_0 (with exponential size)
  - check that it is a solution in exponential time (using automaton)

Constructing a finite-memory strategy

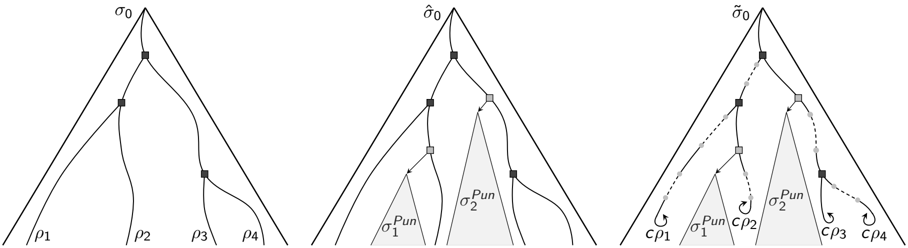
- Given a solution σ_0, take one play ρ_i (witness) for each Pareto-optimal payoff $p_i \in P_{\sigma_0}$



NEXPTIME-membership

Constructing a finite-memory strategy

- Given a solution σ_0, take one play ρ_i (witness) for each Pareto-optimal payoff $p_i \in P_{\sigma_0}$
- Modify σ_0 into $\hat{\sigma}_0$ on deviations from the witnesses: punish by imposing Ω_0 or a not Pareto-optimal payoff
- Modify $\hat{\sigma}_0$ into $\tilde{\sigma}_0$: decompose each ρ_i into at most exponentially many parts and compact it as $c\rho_i$



NP-hardness for reachability objectives on tree arenas

Idea of the proof: NP-hardness is shown using the set cover problem

Given

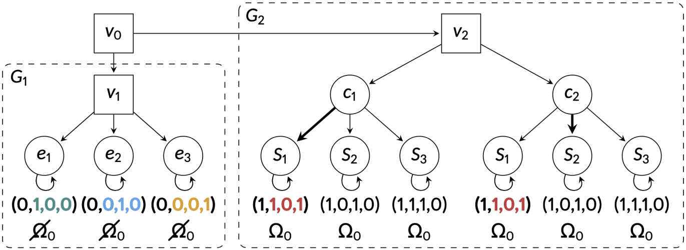
- $C = \{e_1, e_2, \ldots, e_n\}$ of n elements
- m subsets $S_1, S_2, \ldots, S_m$ such that $S_i \subseteq C$
- an integer $k \leq m$

Find k indexes $i_1, i_2, \ldots, i_k$ such that $C = \cup_{j=1}^{k} S_{i_j}$.

Devise a Stackelberg-Pareto game such that Player 0 has a solution to the SPS problem $\Leftrightarrow$ solution to the set cover problem

NP-hardness for reachability objectives on tree arenas

$C = \{e_1, e_2, e_3\}$, $S_1 = \{e_1, e_3\}$, $S_2 = \{e_2\}$, $S_3 = \{e_1, e_2\}$, $k = 2$



- Every play in G_1 is consistent with any strategy of Player 0 and does not satisfy Ω_0
- Hence in a solution, payoffs from G_1 cannot be Pareto-optimal and must be $<$ than some payoff in G_2

Another model - Several followers

Recap

- Environment: one follower with several objectives
- He responds to the announced strategy σ_0 by following a play with Pareto-optimal payoff

Another approach [KPV16, GMP$^+$17]

- Environment: several followers, each with one objective
- Stackelberg game $\mathcal{G} = (G, \Omega_0, \Omega_1, \ldots, \Omega_t)$ with an arena $G = (V, (V_i)_{i=0}^{t}, E, v_0)$, a set $\Pi = \{0, 1, \ldots, t\}$ of players, and an objective Ω_i for Player i, $i \in \Pi$
- These players respond to σ_0 with a strategy profile that is an equilibrium with respect to their own objectives
- Equilibrium: Nash equilibrium, subgame-perfect equilibrium, ...

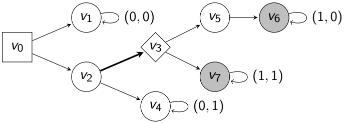
Nash equilibrium

Let σ_0 be a strategy for Player 0

- A σ_0-Stackelberg profile is a strategy profile $\sigma = (\sigma_0, (\sigma_i)_{i \in \Pi \setminus \{0\}})$ such that $pay_i(\langle \sigma \rangle) \geq pay_i(\langle \sigma'_i, \sigma_{-i} \rangle)$ for all players $i \in \Pi \setminus \{0\}$ and all strategies σ'_i for Player i where
  - $\langle \sigma \rangle$ is the play consistent with all strategies of σ
  - $\langle \sigma'_i, \sigma_{-i} \rangle$ is the play consistent with all strategies of σ, except that σ'_i replaces σ_i
- No player $i \neq 0$ has an incentive to deviate from σ_i in a way to increase his payoff

Example

- Player 0: circle vertices
- Player 1: square vertices
- Player 2: diamond vertices
- Strategy σ_0: choice of $v_2 \rightarrow v_3$



Nash equilibrium

Rational synthesis problem (RS problem)

Given a Stackelberg game $\mathcal{G} = (G, \Omega_0, \Omega_1, \ldots, \Omega_t)$, decide whether there exists a strategy σ_0 for Player 0 such that for every σ_0-Stackelberg profile σ, it holds that $\langle \sigma \rangle \in \Omega_0$

Rational verification problem (RV problem)

Given a Stackelberg game $\mathcal{G} = (G, \Omega_0, \Omega_1, \ldots, \Omega_t)$ where the strategy σ_0 of Player 0 is fixed, decide whether for every σ_0-Stackelberg profile σ, it holds that $\langle \sigma \rangle \in \Omega_0$

Results

Theorem

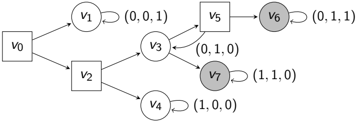
For Stackelberg games

- with LTL objectives, the RS problem is 2EXPTIME-complete [KS22] as well as the RV problem [GNPW20]
- with parity objectives, the RS problem is in EXPTIME and PSPACE-hard [CFGR16] and the RV problem is co-NP-complete [Umm08]

Additional results for subgame perfect equilibria (instead of NEs) in [KPV16, BRvdB22]

Conclusion

- Classical reactive synthesis
  - Model of two-player zero-sum games
  - System and environment have opposed objectives
- Model of Stackelberg non zero-sum games with one follower



  - Verification and synthesis
  - Complexity class and fixed-parameter complexity for ω-regular objectives
- Model of Stackelberg non zero-sum games with several followers

Thanks for your attention!

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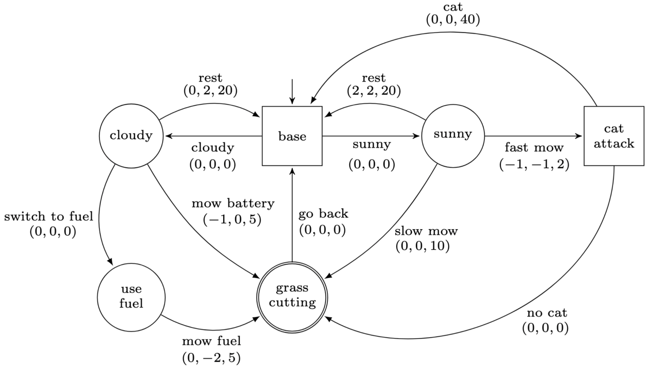
Example

Autonomous robotized lawnmower [Ran12] 

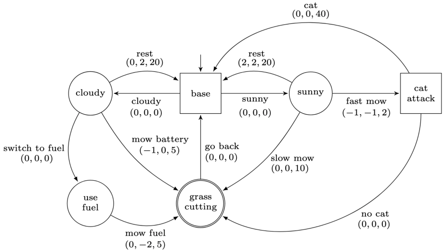
- System: lawnmower with solar panels and fuel tank
- Environment: weather and cat



Example

Objectives

- Büchi objective: grass must be cut infinitely often
- Energy objective: battery and fuel must never drop below 0
- Mean-payoff objective: average time per action must be less than 10 in the long run



Controller as the following strategy

- If sunny, mow slowly
- If cloudy
  - If solar battery ≥ 1, mow on battery
  - otherwise, if fuel level ≥ 2, mow on fuel
  - otherwise, rest at the base